

Constituent and current quark masses at low chiral energies

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Abstract. – Light constituent quark masses and the corresponding dynamical quark masses are determined by data, the Quark-Level Linear σ Model, and infrared QCD. This allows to define effective nonstrange and strange current quark masses which reproduce the experimental pion and kaon masses very accurately, by simple additivity. Moreover, the masses of the light scalar mesons $\sigma(600)$ and $\kappa(800)$ can be obtained straightforwardly from the constituent quark masses. In contrast, the usual nonstrange and strange current quark masses employed by Chiral Perturbation Theory do not allow a simple quantitative explanation of the pion and kaon masses.

Introduction. – In the chiral limit (CL), both the kaon and pion masses vanish. But away from the CL, the chiral-symmetry breaking (ChSB) pion and kaon masses are measured as [1] $m_{\pi^\pm} = 139.57$ MeV, $m_{\pi^0} = 134.98$ MeV, $m_{K^\pm} = 493.677$ MeV and $m_{K^0} = 497.648$ MeV (dropping the small experimental errors). The kaon $\bar{q}q$ mass difference is in fact [2, 3]

$$m_{K^0}(\bar{s}d) - m_{K^+}(\bar{s}u) = (m_d - m_u)_{\text{cur}} = (497.648 - 493.677) \text{ MeV} = 3.97 \text{ MeV}, \quad (1)$$

where the strange-current-quark-mass component cancels out. Likewise, the difference of baryon masses composed of qqq constituent quarks for Σ^- and Σ^+ is [1, 2]

$$m_{\Sigma^-(sdd)} - m_{\Sigma^+(suu)} = 8.08 \text{ MeV} \\ \Rightarrow (m_d - m_u)_{\text{con}} = 4.04 \text{ MeV}, \quad (2)$$

where the strange-constituent-quark-mass component also cancels out.

Since the strong-interaction bulk dynamical quark mass $m_{\text{dyn}} = m_{\text{con}} - m_{\text{cur}}$ conserves isospin, it should not be surprising that at low energies both the constituent and current quark-mass differences are [3]

$$(m_d - m_u)_{\text{con}} = (m_d - m_u)_{\text{cur}} \approx 4 \text{ MeV}, \quad (3)$$

compatible with Eqs. (1,2) above. Moreover, a bulk dynamical CL nonstrange quark mass can be estimated as

$$m_{\text{dyn}} \approx \frac{m_N}{3} \approx 313 \text{ MeV}, \quad (4)$$

while the CL pion charge radius $r_\pi^{\text{CL}} \approx 0.63$ fm (as found from vector-meson dominance [4]) implies

$$m_{\text{dyn}} \approx \frac{\hbar c}{r_\pi^{\text{CL}}} = \frac{197.3 \text{ MeV} \cdot \text{fm}}{0.63 \text{ fm}} = 313 \text{ MeV}. \quad (5)$$

Lastly, low-energy QCD requires

$$m_{\text{dyn}} = \left[\frac{4\pi}{3} \alpha_s \langle -\bar{q}q \rangle \right]^{\frac{1}{3}}. \quad (6)$$

Then, demanding the latter to equal again 313 MeV in turn suggests [5, 6]

$$\alpha_s(1 \text{ GeV}) \approx 0.50, \quad \langle -\bar{q}q \rangle^{\frac{1}{3}} \approx 245 \text{ MeV}. \quad (7)$$

This further suggests [6] a meson-quark coupling, for $N_c = 3$,

$$g_{\pi qq} = \frac{2\pi}{\sqrt{3}}, \quad \alpha_s^{\text{eff}} = \frac{g^2}{4\pi} = \frac{\pi}{3} \sim 1, \quad (8)$$

so that the scalar σ meson resonance condenses at a value twice that of the average constituent quark mass given by the nucleon magnetic moment in the nonrelativistic quark model (NRQM), viz. [3]

$$\hat{m}_{\text{con}} = \frac{1}{2}(m_u + m_d)_{\text{con}} \approx 337.5 \text{ MeV}. \quad (9)$$

This occurs at a scalar mass $m_\sigma \approx 630$ – 660 MeV. Note that Eq. (9) is very near the constituent mass resulting

from the quark-model chiral Goldberger–Treiman relation [3, 7], i.e.,

$$\hat{m}_{\text{con}} = f_\pi g_{\pi qq} \approx 93 \text{ MeV} \times \frac{2\pi}{\sqrt{3}} \approx 337.4 \text{ MeV} , \quad (10)$$

with

$$m_\sigma = 2m_{\text{dyn}} \approx 630 \text{ MeV} \quad (11)$$

in the CL.

Effective current quark mass in QCD. – Away from the CL, the effective current quark mass is

$$\hat{m}_{\text{cur}} = \hat{m}_{\text{con}} - m_{\text{dyn}} , \quad (12)$$

where m_{dyn} in QCD runs as

$$m_{\text{dyn}}(p^2) \propto p^{-2} . \quad (13)$$

On the $\hat{m}_{\text{con}} \approx 337.5 \text{ MeV}$ mass shell, self-consistency then requires

$$m_{\text{dyn}}(p^2 = \hat{m}^2) = \frac{m_{\text{dyn}}^3}{\hat{m}^2} = \frac{(313)^3}{(337.5)^2} \text{ MeV} = 269.2 \text{ MeV} , \quad (14)$$

generating an effective current quark mass, via Eq. (12),

$$\hat{m}_{\text{cur}}^{\text{eff}} = (337.5 - 269.2) \text{ MeV} = 68.3 \text{ MeV} . \quad (15)$$

The latter is very near the ChSB pion-nucleon sigma term, from different analyses:

$$\begin{cases} \sigma_{\pi N} = (55 \pm 13) \text{ MeV} [8] , \\ \sigma_{\pi N} = (66 \pm 9) \text{ MeV} [9] , \\ \sigma_{\pi N} = (64 \pm 8) \text{ MeV} [10] . \end{cases} \quad (16)$$

Pion $\bar{q}q$ mass. – Given $\hat{m}_{\text{cur}}^{\text{eff}}$ from Eq. (15), the ChSB $\bar{q}q$ pion mass is [2, 5]

$$\bar{m}_\pi = 2\hat{m}_{\text{cur}}^{\text{eff}} = 136.6 \text{ MeV} , \quad (17)$$

almost midway between the observed [1] $m_{\pi^0} = 134.98 \text{ MeV}$ and $m_{\pi^\pm} = 139.57 \text{ MeV}$. In fact, the ChSB isovector pion masses $m_{\pi^\pm} = \bar{m}_\pi + 2\varepsilon$ and $m_{\pi^0} = \bar{m}_\pi - \varepsilon$ have average value

$$\bar{m}_\pi = \frac{1}{3}(m_{\pi^\pm} + 2m_{\pi^0}) = 136.5 \text{ MeV} , \quad (18)$$

very near the above estimate of 136.6 MeV.

Kaon $\bar{q}q$ masses. – Given $m_d - m_u \approx 4 \text{ MeV}$ and neglecting small experimental errors, the observed ChSB kaon masses are

$$\begin{aligned} m_{K^+(\bar{s}u)} &= m_{s,\text{cur}} + m_{u,\text{cur}} = \\ &\hat{m}_{\text{cur}} \left[1 + \left(\frac{m_s}{\hat{m}} \right)_{\text{cur}} \right] - 2 \text{ MeV} = 493.677 \text{ MeV} , \\ m_{K^0(\bar{s}d)} &= m_{s,\text{cur}} + m_{d,\text{cur}} = \\ &\hat{m}_{\text{cur}} \left[1 + \left(\frac{m_s}{\hat{m}} \right)_{\text{cur}} \right] + 2 \text{ MeV} = 497.648 \text{ MeV} . \end{aligned} \quad (19)$$

For $\hat{m}_{\text{cur}}^{\text{eff}}$, this gives in *both* cases, for $m_\pi = 2\hat{m}_{\text{cur}}$,

$$\left(\frac{m_s}{\hat{m}} \right)_{\text{cur}} = \frac{2 \times 495.7 \text{ MeV}}{136.6 \text{ MeV}} - 1 \approx 6.26 , \quad (20)$$

taking the average kaon mass as $(m_{K^+} + m_{K^0})/2 = 495.7 \text{ MeV}$. The latter ratio compares well with the light-plane [11] and $\pi\pi$ [12] predictions 6–7 and 6.33, respectively.

Independently of the above ratios 6.26, 6–7, 6.33, the “good-bad” chiral operators lead to the infinite-momentum-frame (IMF) estimate [13]

$$\left(\frac{m_s}{\hat{m}} \right)_{\text{cur}}^{\text{IMF}} = \left[\left(\frac{2m_K^2}{\hat{m}^2} \right) - 1 \right]^{\frac{1}{2}} \approx 5 , \quad (21)$$

requiring both squared meson and current quark masses in the IMF.

$(m_s/\hat{m})_{\text{cur}}$ from nonrenormalisation theorem. – Dealing instead with chiral current algebra, the vector-current divergence $i\partial V^{6+i7}$ generates [14]

$$\sqrt{2}\langle \pi^0 | i\partial V^{6+i7} | \bar{K}^0 \rangle = f_+(0)(m_K^2 - m_\pi^2) , \quad (22)$$

for the nonrenormalisation-theorem [15] value [16]

$$f_+(0) = 1 - \frac{g_{\pi qq}^2 \delta^2}{8\pi^2} \approx 0.9677 , \quad (23)$$

for $\delta = (m_s/\hat{m})_{\text{con}} - 1 \approx 0.44$. Then, with $f_K/f_\pi \approx 1.22$ [1] and $f_K/(f_\pi f_+(0)) \equiv X \approx 1.2607$, Eqs. (22,23) generate the current-quark-mass ratio [14] (for $\bar{m}_\pi^2/\bar{m}_K^2 \approx 0.07594$)

$$\left(\frac{m_s}{\hat{m}} \right)_{\text{cur}} = \frac{X + 1 - \frac{\bar{m}_\pi^2}{\bar{m}_K^2}}{X - 1 + \frac{\bar{m}_\pi^2}{\bar{m}_K^2}} \approx 6.49 . \quad (24)$$

The latter ratio is quite near the values 6–7, 6.26, 6.33 and 5 found above. Note that, as $X \rightarrow 1$, $(m_s/\hat{m})_{\text{cur}} \rightarrow 2\bar{m}_K^2/\bar{m}_\pi^2 - 1$, that is, the canonical chiral-perturbation-theory (ChPT) value (see Eq. (34) below).

The current-divergence procedure leading to the ratio in Eq. (24) can be extended to the constituent-quark decuplet-baryon mass difference and current-quark mass difference in $\Xi^-(ssd) - \Xi^0(ssu)$, using $SU(6)$ d/f ratios [17]. In any case, a slightly larger σ -resonance mass (650–670 MeV) is obtained with standard dispersion theory and quark-level-linear- σ -model (QLL σ M) [18] techniques, but *not* using 1- or 2-loop ChPT [19–21].

Pion-nucleon σ -term. – Specifically, ChPT before 1982 [19] suggested that the πN σ -term was near the Gell-Mann–Oakes–Renner (GMOR) [22] value of 26 MeV, and then the observed σ resonance was unrelated to the original ChPT (see Ref. [23], App. B). However, in 1991 ChPT instead claimed a πN σ -term of 60 MeV follows from the positive and coherent sum of *four* terms [20, 21], at the Cheng–Dashen (CD) [24] point $t = 2m_\pi^2$ (in MeV), i.e.,

$$\begin{aligned} \sigma_{\pi N}(t = 2m_\pi^2) &= \sigma_{\pi N}^{\text{GMOR}} + \sigma_{\pi N}^{\text{HOChPT}} + \sigma_{\pi N}^{\bar{s}s} + \sigma_{\pi N}^{t\text{-dep.}} \\ &\approx (25 + 10 + 10 + 15) \text{ MeV} = 60 \text{ MeV} . \end{aligned} \quad (25)$$

Here, the second term on the right-hand side arises from higher-order ChPT, the third one from the strange-quark sea, and the fourth is a t -dependent contribution due to going from $t = 0$ to the CD point $t = 2m_\pi^2$, where the πN background is minimal. Leutwyler [21] concluded: “*The three pieces happen to have the same sign.*” Of course, for things to work out right, all *four* pieces must have the same sign, including the GMOR term, which reads

$$\sigma_{\pi N}^{\text{GMOR}} = \frac{m_\Xi + m_\Sigma - 2m_N}{2} \left(\frac{m_\pi^2}{m_K^2 - m_\pi^2} \right) \approx 26 \text{ MeV}. \quad (26)$$

We prefer instead to invoke the model-independent IMF version, which has only *one* net term, viz.

$$\sigma_{\pi N}^{\text{IMF}} = \frac{m_\Xi^2 + m_\Sigma^2 - 2m_N^2}{2m_N} \left(\frac{m_\pi^2}{m_K^2 - m_\pi^2} \right) \approx 63 \text{ MeV}, \quad (27)$$

which is compatible with observation [8–10], and also with the ChSB effective current quark mass of about 68 MeV as found in Eq. (15) above.

Strange current and constituent quark masses. – Given the average kaon mass and Eq. (1) — but now with a plus sign — we get for a $\bar{q}q$ kaon

$$\overline{m}_K = \frac{1}{2} [m_{K^0}(\bar{s}d) + m_{K^+}(\bar{s}u)] \approx 495.7 \text{ MeV} \\ = (m_s + \hat{m})_{\text{cur}}. \quad (28)$$

Subtracting now from Eq. (28) the nonstrange current quark mass $\hat{m}_{\text{cur}}^{\text{eff}} = 68.3 \text{ MeV}$ derived in Eq. (15), we deduce a strange current quark mass of

$$m_{s,\text{cur}} = (495.7 - 68.3) \text{ MeV} = 427.4 \text{ MeV}, \quad (29)$$

which is of course very near the current-quark-ratio version

$$m_{s,\text{cur}} = \left(\frac{m_s}{\hat{m}} \right)_{\text{cur}} \times \hat{m}_{\text{cur}} \approx 6.26 \times 68.3 \text{ MeV} = 427.5 \text{ MeV}. \quad (30)$$

As for the strange constituent quark mass, we may obtain an estimate using the chiral quark-level Goldberger–Treiman relation for *constituent* quarks:

$$\left. \begin{aligned} \frac{1}{2}(m_s + \hat{m})_{\text{con}} &= f_K g_{Kqq}, \quad g_{Kqq} = g_{\pi qq} \quad [3], \quad \text{or} \\ \left(\frac{m_s + \hat{m}}{2\hat{m}} \right)_{\text{con}} &= \frac{f_K}{f_\pi} \approx 1.22 \quad [1] \end{aligned} \right\} \Rightarrow \\ m_{s,\text{con}} = 1.44 \hat{m}_{\text{con}} = 1.44 \times 337.5 \text{ MeV} \approx 486 \text{ MeV}, \quad (31)$$

near the strange-quark valence value of 515 MeV [25].

Alternatively, let us estimate $m_{s,\text{con}}$ from the Σ baryon mass as

$$\frac{m_{\Sigma^-} + m_{\Sigma^+}}{2} = 1193 \text{ MeV} = m_{s,\text{con}} + 2\hat{m}_{\text{con}}, \quad (32)$$

which yields $m_{s,\text{con}} \approx 518 \text{ MeV}$, close to the values 515 MeV and 486 MeV above. A similar constituent strange quark mass also follows from the vector-meson $\bar{s}s$ $\phi(1020)$ mass, suggesting $m_{s,\text{con}} \approx 1020/2 \text{ MeV} = 510 \text{ MeV}$.

$(m_s/\hat{m})_{\text{cur}}$ in ChPT. – Alternatively, we may compare this with the ChPT prediction for the current quark mass (see e.g. review from 1982 [19]), for $f_\pi \approx 93 \text{ MeV}$,

$$\hat{m}_{\text{cur}} = \frac{(f_\pi m_\pi)^2}{2\langle -\bar{q}q \rangle} \approx 5.6 \text{ MeV}, \quad (33)$$

again using $\langle -\bar{q}q \rangle^{1/3} \approx 245 \text{ MeV}$. When this is combined with the ChPT ratio [19]

$$\left(\frac{m_s}{\hat{m}} \right)_{\text{cur}} \approx \frac{2m_K^2}{m_\pi^2} - 1 \approx 25.3, \quad (34)$$

one could infer a strange current quark mass of

$$m_{s,\text{cur}} \approx 25.3 \times 5.6 \text{ MeV} \approx 142 \text{ MeV}, \quad (35)$$

so only about 29% of the strange constituent quark mass of 486 MeV found in Eq. (31), which does not seem very realistic. This is to be contrasted with $m_{s,\text{cur}}/m_{s,\text{con}} \approx 88\%$ from our Eq. (29) above.

Studying the ChPT Eqs. (33,34) independently of one another, the ChSB nonstrange relation in Eq. (33) appears to be unrelated to the observed ChSB πN σ -terms in Refs. [8–10], and also to the observed $\bar{q}q$ pion mass of 138 MeV, which we managed to simply relate to the effective nonstrange current quark mass in Eqs. (15,17) as $m_\pi \approx 2m_{\text{cur}}^{\text{eff}}$. Moreover, the ratio in Eq. (34) is about a factor 4 larger than the values 6.26 from the average kaon mass in Eq. (20), 6–7 from the light plane [11], 6.33 from $\pi\pi$ scattering [12], and 6.49 from the current-divergence nonrenormalisation relation (24).

Finally, we refer to an unpublished paper from 1974 [26] also finding a current-quark-mass ratio $(m_s/\hat{m})_{\text{cur}} \sim 6$, which avoided “bad” quark operators such as $\bar{q}\lambda_q\gamma_5 q$ in Ref. [22]. Note that in the latter reference, GMOR suggested $f_K = f_\pi$ to first order away from the CL. We argue that the ChSB theory as employed in Refs. [19–21, 27] must invoke a 22% ChSB increase even in first order, because data says [1] $f_K/f_\pi \approx 1.22$, *not* $f_K \approx f_\pi$.

With hindsight, Ref. [26] appears to realise that the standard perturbative Taylor-series approach to the ratio of strange over nonstrange current quark mass *breaks down* when $f_K \rightarrow f_\pi$, as we have shown in Eq. (24), which does *not* give Eq. (34) as ChPT suggests. Even *generalised* ChPT admits the possibility that this ratio may be considerably smaller [28] than the standard ChPT value ~ 25 .

We are well aware that recent lattice analyses report values for the ratio m_s/\hat{m} close to the ChPT prediction, namely 25.7 (CP-PACS and JLQCD Collaborations) [29], 27.2 (HPQCD Collaboration) [30], 25.3 (QCDSF Collaboration) [31] and 27.4 (MILC, HPQCD and UKQCD Collaborations) [32]. However, this is not so surprising, as all these lattice results, obtained at a scale of 2 GeV in the $\overline{\text{MS}}$ scheme, strongly rely upon perturbative chiral extrapolations in the framework of *standard* ChPT. Such extrapolations assume the validity of the GMOR relation, ruling out significant corrections to it, in contrast with approaches like *generalized* ChPT and also the QLL σ M. Let

us furthermore quote from the most recent of these lattice works, namely Ref. [29]: “*We note that our WChPT fits to data do not exhibit a clear chiral logarithm, probably because u and d quark masses in our simulation are not sufficiently small.*” In view of these and other uncertainties, like finite-volume effects, we believe there is no conflict between the very small current quark masses found in such lattice/ChPT approaches and the *effective* current quark masses in the present paper.

Scalar-meson masses and summary. — With regard to the nonstrange and strange constituent quark masses, we may estimate the corresponding scalar-meson masses m_σ and m_κ , where σ and κ stand for the PDG states $f_0(600)$ and $K_0^*(800)$ [1], respectively. For the σ , we get [5]

$$m_\sigma \approx 2\hat{m}_{\text{con}} \approx 625\text{--}675 \text{ MeV} , \quad (36)$$

depending on whether we work in the CL for \hat{m}_{con} or not. Regarding the κ , the resulting mass range is [5]

$$m_\kappa \approx 2\sqrt{m_{s,\text{con}}\hat{m}_{\text{con}}} \approx 780\text{--}810 \text{ MeV} . \quad (37)$$

The latter range is entirely compatible with the E791 [33] value

$$m_\kappa^{\text{E791}} = (791 \pm 19) \text{ MeV} , \quad (38)$$

found from $D^+ \rightarrow K^-\pi^+\pi^+$ decay, and also with the resonance peak masses obtained in the unitarised quark-meson models of Ref. [34].

ChPT, on the other hand, has a much more complicated relationship with the light scalar mesons. During many years, ChPT disputed the relevance or even the very existence of the σ meson. However, recently [27] Roy equations were employed to generate a σ pole, which indeed is in rough agreement with Weinberg’s [35] estimate $\Gamma_\sigma \approx 9\Gamma_\rho/2$. These equations amount to twice-subtracted dispersion relations in which the two subtraction constants are fixed via higher-order ChPT. While such a unitarisation of ChPT “by hand” is not really new (see e.g. Ref. [36]), completely unprecedented is the claim to have pinned down the σ pole position with surprisingly small error bars. However, the latter cannot be trusted in view of the neglected σ - $f_0(980)$ mixing and the deficient treatment of the $\bar{K}K$ channel [37]. Moreover, the Roy-equation approach as applied in Ref. [27] may be at odds [38] with the standard Chew-Mandelstam [39] double-dispersion representation of $\pi\pi$ amplitudes [18]. For all that, even the scattering lengths entering the Roy equations are to be questioned, as they are related to scalar radii which yield ChSB effects of the order of 6–8%, instead of the observed 3%.

In summary, the present paper shows how constituent, dynamical and current quark masses — compatible with QCD expectations — can be defined that are related in a very simple way to light meson masses and other low-energy observables. In particular, the thus determined effective nonstrange and strange current quark masses

determine the pion and kaon masses just by additivity. Moreover, the latter nonstrange current quark mass is of the same size as another crucial measure of ChSB, viz. the πN σ -term. Finally, our constituent quark masses allow to straightforwardly obtain the light scalar meson masses m_σ and m_κ in agreement with experiment. In contrast, ChPT does not allow any simple relation between quark masses and light-meson masses or the πN σ -term.

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